Explicit Analytical Relations for Strain-controlled Rheodynamical Quantities in the Case of Zener-Arrhenius Model
I. Underlying relations

HORIA PAVEN*

National Institute of Research and Development for Chemistry and Petrochemistry - ICECHIM, 202 Splaiul Independentei, 060021, Bucharest, Romania

In order to point out the intrinsic peculiarities of thermorheodynamical behaviour of solid-like polymer materials, the framework of a Zener - Arrhenius model is considered, the frequency or/and temperature dependences being approached in the case of strain-controlled conditions. The complete set of the explicit general relations providing the characteristic thermorheodynamical quantities (the storage, loss and absolute moduli, as well as the loss factor, and the corresponding storage, loss and absolute compliances) is presented in terms of meaningful rheological parameters including the low - and high frequency limit values of the storage modulus, and the appropriate relaxation time. The underlying analytical relations are given in forms suitable for direct numerical simulation of both isothermal frequency - dependence, and well as of isochronal temperature - dependence circumstances.

Keywords: analytical relations, characteristic thermorheodynamical quantities, strain-controlled conditions, Zener - Arrhenius model, isothermal and isochronal circumstances.

The ability of solid-like polymer materials to specifically deform is a direct consequence of intrinsic mobility of their molecular entities to accomplish typical motions including, in the case of strain-controlled conditions, the relaxation ones. Moreover, given the influence of the dynamic deformation-driven level, as well as of those of frequency and temperature dependences, the viscoelastic-like effects are properly evidenced by the rheological quantities [1-5].

In fact, linear viscoelasticity allows for time dependence in the rheological stress-strain relationships by considering a linear function of stress and its derivatives with respect to time to be equated to a linear function of strain and its time derivatives [6-11].

Accordingly, the present article intend to clarify the essentials of different relationships expressing the frequency and temperature variation trends of rheodynamical quantities, and to provide an appropriate characterization of the linear viscoelastic peculiarities of rheological behaviour by using a "relaxation and corresponding retardation" description in the framework of a coupled Zener - Arrhenius model. Thus, the consequences of the explicit general and characteristic forms of the complete set of linear viscoelastic rheodynamical quantities point out the frequency or/and temperature dependences in the case of a sinusoidal strain-controlled excitation, the underlying relations being considered both in terms of phenomenological and physico-chemical meaningful parameters.

The proposed approach supposes the consideration of the typical rheological equation

$$p_0(T)\sigma + p_1(T)D_t\sigma = q_0(T)\varepsilon + q_1(T)D_t\varepsilon \qquad (1)$$

in terms of ε and σ (the strain and stress, respectively), where p_0, p_1 and q_0, q_1 are the nominal temperature dependent rheological parameters, the time contribution being stated by means of time derivative term, $D_t = \partial/\partial t$.

As a matter of fact, it results that this model is a three - parameter one, i. e.,

$$[1 + B_1(T)D_t]\sigma = [A_0(T) + A_1(T)D_t]\varepsilon \qquad (2.1)$$

with the corresponding characteristic rheological parameters $A_0(T)$, $A_1(T)$, $B_1(T)$ given by

$$A_0(T) = \frac{q_0(T)}{p_0(T)}, \ A_1(T) = \frac{q_1(T)}{p_0(T)}, \ B_1(T) = \frac{p_1(T)}{p_0(T)} \qquad (3)$$

Taking into account the intrinsic linearity of considered rheological equation from the viewpoint of time-derivative operator, D_t, in case of a controlled sinusoidal dynamic strain,

$$\varepsilon \equiv \widetilde{\varepsilon} = \varepsilon_0 e^{i\omega t} \qquad (4)$$

where ε_0 is the strain amplitude and ω - the angular frequency of the given sinusoidal strain - the corresponding dynamic form results as ($D_t = i\omega$)

$$[1 + i\omega B_1(T)]\sigma = [A_0(T) + i\omega A_1(T)]\varepsilon \qquad (2.2)$$

The involved stress - with the resulting amplitude σ_0, and of same frequency as that of the strain and, as result of internal friction, with a phase lag, δ, between the excitation and response - is

$$\sigma = \widetilde{\sigma} = \sigma_0(\omega,T)e^{i[\omega t + \delta_M(\omega,T)]} \qquad (5)$$

Accordingly, by definition

$$\frac{\widetilde{\sigma}}{\widetilde{\varepsilon}} = \frac{\sigma_0(\omega,T)}{\varepsilon_0}[\cos\delta_M(\omega,T) + i\sin\delta_M(\omega,T)] =$$
$$= \widetilde{M}(i\omega,T) \equiv M^*(\omega,T) = M'(\omega,T) + iM''(\omega,T) \qquad (6)$$

* email: htopaven@netscape.net; Tel.: 0726394575

is the complex modulus, M' and M" being the storage and loss moduli, respectively. On the other hand, for the absolute (viscoelastic) modulus, $|M^*|$, results

$$|M^{\bullet}(\omega,T)|=[M'^2(\omega,T)+iM''^2(\omega,T)]^{1/2} \qquad (7)$$

whereas $\beta_M = \tan \delta_M$ stands for the corresponding loss factor, being given as

$$\beta_M(\omega,T)=\tan\delta_M(\omega,T)=M''(\omega,T)/M'(\omega,T) \qquad (8)$$

From a "hierarchical" viewpoint, it appears that in the case of ε - controlled excitation, M', M" are the *direct primary* rheological quantities, whereas $|M^*|$, β_M are the *derivate primary* ones. Moreover, the complete set of ε rheological quantities also includes the well defined compliance-like quantities - the corresponding storage compliance, J'$_M$,

$$J'_M(\omega,T)=M'(\omega,T)/|M^{\bullet}(\omega,T)|^2 \qquad (9)$$

loss compliance, J"$_M$,

$$J''_M(\omega,T)=M''(\omega,T)/|M^{\bullet}(\omega,T)|^2 \qquad (10)$$

and absolute compliance, $|J^*_M|$,

$$|J^{\bullet}_M(\omega,T)|=[J'^2_M(\omega,T)+J''^2_M(\omega,T)]^{1/2} \qquad (11)$$

which are termed as *secondary* rheodynamical quantities.

In order to include, besides the frequency contribution, that of temperature one - it is suggested that, in principle, the deformation rate could be expressed by an Arrhenius-like relation [12-17], the most frequently used form, at least as a preliminary evaluation tool, being an exponential one defined for the ε –controlled relaxation time, τ_ε, as

$$\tau_\varepsilon(T)=\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{T}} \qquad (12)$$

where $\tau_{\varepsilon\infty}=\tau_\varepsilon(\infty)$ is the prefactor (pre-exponential factor) - denoting the value of the corresponding relaxation time at infinite temperature (if $\theta_\varepsilon \neq 0$), while $\theta_\varepsilon = A_\varepsilon/R$ - i. e., the ratio of the ε − Arrhenius activation energy, A_ε, to the universal gas constant, R = 8.314 J / (mol* K) - points a "virtual" ε temperature.

Consequently, the complete "frequency and temperature" form of rheodynamic equation arises as

$$[1+i\omega\tau_\varepsilon(T)]\sigma=[M'_l+i\omega M'_h\tau_\varepsilon(T)]\varepsilon \qquad (2.3)$$

where

$$M'_l \equiv M'(0)=A_0, M'_h \equiv M'(\infty)=\frac{A_1}{B_1}, \tau_\varepsilon=B_1 (13)$$

the meaning of different rheological parameters being expressed in terms of the storage modulus at low (l) and high limit (h) frequency, respectively, and the ε − characteristic relaxation time.

Results and discussions

As results from the general form of the dynamic rheological equation (2.3), there are distinct dependences of characteristic rheodynamical quantities on frequency and temperature, respectively. Thus, for the complete set of seven above quoted rheological quantities, two circumstances are to be taken into account - on the hand,

that of frequency dependence at given temperature, which corresponds to *isothermal* circumstances, and, on the other hand, that of temperature dependence, when the data are obtained in *isochronal* circumstances.

By using the general definition of temperature and frequency dependences of rheodynamic quantities, the characteristic relations are indicated by a semicolon (;), where the first term denotes the continuous variable, while the second one appears as a parameter.

In the case of a sinusoidal strain-controlled excitation, the complete set of primary and secondary rheodynamic quantities includes the corresponding general and characteristic forms as follows.

Direct primary quantities
- The general form of the *storage modulus* results, on the basis of (2), (6) and (13), as

$$M'(\omega,T)=\frac{A_0(T)+A_1(T)B_1(T)\omega^2}{1+B_1^2(T)\omega^2}=$$

$$=\frac{M'_l(T)+M'_h(T)\tau_{\varepsilon\infty}^2 e^{\frac{2\theta_\varepsilon}{T}}\omega^2}{1+\tau_{\varepsilon\infty}^2 e^{\frac{2\theta_\varepsilon}{T}}\omega^2} \qquad (14)$$

Accordingly, for the frequency dependence at given temperature, T = $\underline{T}$, one obtains

$$M'(\omega;\underline{T})=\frac{A_0(\underline{T})+A_1(\underline{T})B_1(\underline{T})\omega^2}{1+B_1^2(\underline{T})\omega^2}=\frac{A_{0\underline{T}}+A_{1\underline{T}}B_{1\underline{T}}\omega^2}{1+B_{1\underline{T}}^2\omega^2}=$$

$$=\frac{M'_{l\underline{T}}+M'_{h\underline{T}}\tau_{\varepsilon\infty}^2 e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2}{1+\tau_{\varepsilon\infty}^2 e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2} \qquad (14.1)$$

and

$$M'(T;\underline{\omega})=\frac{A_0(T)+\underline{\omega}^2 A_1(T)B_1(T)}{1+\underline{\omega}^2 B_1^2(T)}=$$

$$=\frac{M'_l(T)+\underline{\omega}^2 M'_h(T)\tau_{\varepsilon\infty}^2 e^{\frac{2\theta_\varepsilon}{T}}}{1+\underline{\omega}^2\tau_{\varepsilon\infty}^2 e^{\frac{2\theta_\varepsilon}{T}}} \qquad (14.2)$$

in the case of temperature dependence at given frequency, $\omega = \underline{\omega}$.
- The general expression of the *loss modulus* is, by taking into account (2), (6) and (13),

$$M''(\omega,T)=\frac{[A_1(T)-A_0(T)B_1(T)]\omega}{1+B_1^2(T)\omega^2}=$$

$$=\frac{[M'_h(T)-M'_l(T)]\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{T}}\omega}{1+\tau_{\varepsilon\infty}^2 e^{\frac{2\theta_\varepsilon}{T}}\omega^2} \qquad (15)$$

Correspondingly, for the ω - dependence at fixed temperature, T=$\underline{T}$,

$$M''(\omega;\underline{T})=\frac{[A_1(\underline{T})-A_0(\underline{T})B_1(\underline{T})]\omega}{1+B_1^2(\underline{T})\omega^2}=\frac{(A_{1\underline{T}}-A_{0\underline{T}}B_{1\underline{T}})\omega}{1+B_{1\underline{T}}^2\omega^2}=$$

$$=\frac{(M'_{h\underline{T}}-M'_{l\underline{T}})\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{\underline{T}}}\omega}{1+\tau_{\varepsilon\infty}^2 e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2} \qquad (15.1)$$

and

$$M''(T;\underline{\omega}) = \underline{\omega}\frac{[A_1(T) - A_0(T)B_1(T)]}{1 + \underline{\omega}^2 B_1^{\;2}(T)} =$$

$$= \underline{\omega}\frac{[M_h'(T) - M_l'(T)]\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{T}}}{1 + \underline{\omega}^2\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}} \qquad (15.2)$$

in the case of T - dependence at given frequency, $\omega = \underline{\omega}$.

Derivate primary quantities
- The general form of the *absolute modulus* results, by using (2), (8) and (13), as

$$|M^*(\omega,T)| = [\frac{A_0^{\;2}(T) + A_1^{\;2}(T)\omega^2}{1 + B_1^{\;2}(T)\omega^2}]^{1/2} =$$

$$= [\frac{M_l'^{2}(T) + M_h'^{2}(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}\omega^2}{1 + \tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}\omega^2}]^{1/2} \quad (16)$$

Accordingly, for the frequency dependence at given temperature, $T = \underline{T}$, one obtains

$$|M^*(\omega;\underline{T})| = [\frac{A_0^{\;2}(\underline{T}) + A_1^{\;2}(\underline{T})\omega^2}{1 + B_1^{\;2}(\underline{T})\omega^2}]^{1/2} = (\frac{A_{0\underline{T}}^{\;2} + A_{1\underline{T}}^{\;2}\omega^2}{1 + B_{1\underline{T}}^{\;2}\omega^2})^{1/2} =$$

$$= (\frac{M_{l\underline{T}}'^{\;2} + M_{h\underline{T}}'^{\;2}\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2}{1 + \tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2})^{1/2} \qquad (16.1)$$

and

$$|M^*(T;\underline{\omega})| = [\frac{A_0^{\;2}(T) + \underline{\omega}^2 A_1^{\;2}(T)}{1 + \underline{\omega}^2 B_1^{\;2}(T)}]^{1/2} =$$

$$= [\frac{M_l'^{2}(T) + \underline{\omega}^2 M_h'^{2}(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}}{1 + \underline{\omega}^2\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}}]^{1/2} \qquad (16.2)$$

in the case of temperature dependence at given frequency, $\omega = \underline{\omega}$.
- The general expression of the *loss factor* is, given (2), (8), (13),

$$\beta_M(\omega,T) \equiv \tan\delta_M(\omega,T) = \frac{[A_1(T) - A_0(T)B_1(T)]\omega}{A_0(T) + A_1(T)B_1(T)\omega^2} =$$

$$= \frac{[M_h'(T) - M_l'(T)]\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{T}}\omega}{M_l'(T) + M_h'(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}\omega^2} \qquad (17)$$

while for the ω - dependence at fixed temperature, $T = \underline{T}$,

$$\beta_M(\omega;\underline{T}) \equiv \tan\delta_M(\omega;\underline{T}) = \frac{[A_1(\underline{T}) - A_0(\underline{T})B_1(\underline{T})]\omega}{A_0(\underline{T}) + A_1(\underline{T})B_1(\underline{T})\omega^2} =$$

$$= \frac{(A_{1\underline{T}} - A_{0\underline{T}}B_{1\underline{T}})\omega}{A_{0\underline{T}} + A_{1\underline{T}}B_{1\underline{T}}\omega^2} = \frac{(M_{h\underline{T}}' - M_{l\underline{T}}')\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{\underline{T}}}\omega}{M_{l\underline{T}}' + M_{h\underline{T}}'\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2} \quad (17.1)$$

and

$$\beta_M(T;\underline{\omega}) \equiv \tan\delta_M(T;\underline{\omega}) = \underline{\omega}\frac{[A_1(T) - A_0(T)B_1(T)]}{A_0(T) + \underline{\omega}^2 A_1(T)B_1(T)} =$$

$$= \underline{\omega}\frac{[M_h'(T) - M_l'(T)]\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{T}}}{M_l'(T) + \underline{\omega}^2 M_h'(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}}$$

in the case of T - dependence at given frequency, $\omega = \underline{\omega}$.

Coresponding secondary quantities
- The general form of the *corresponding storage compliance* results, on the basis of (2), (9) and (13), as

$$J_M'(\omega,T) = \frac{A_0(T) + A_1(T)B_1(T)\omega^2}{A_0^{\;2}(T) + A_1^{\;2}(T)\omega^2} =$$

$$= \frac{M_l'(T) + M_h'(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}\omega^2}{M_l'^{2}(T) + M_h'^{2}(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}\omega^2} \qquad (18)$$

whereas for the frequency dependence at given temperature, $T = \underline{T}$, one obtains

$$J_M'(\omega;\underline{T}) = \frac{A_0(\underline{T}) + A_1(\underline{T})B_1(\underline{T})\omega^2}{A_0^{\;2}(\underline{T}) + A_1^{\;2}(\underline{T})\omega^2} = \frac{A_{0\underline{T}} + A_{1\underline{T}}B_{1\underline{T}}\omega^2}{A_{0\underline{T}}^{\;2} + A_{1\underline{T}}^{\;2}\omega^2} =$$

$$= \frac{M_{l\underline{T}}' + M_{h\underline{T}}'\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2}{M_{l\underline{T}}'^{\;2} + M_{h\underline{T}}'^{\;2}\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2} \qquad (18.1)$$

and

$$J_M'(T;\underline{\omega}) = \frac{A_0(T) + \underline{\omega}^2 A_1(T)B_1(T)}{A_0^{\;2}(T) + \underline{\omega}^2 A_1^{\;2}(T)} =$$

$$= \frac{M_l'(T) + \underline{\omega}^2 M_h'(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}}{M_l'^{2}(T) + \underline{\omega}^2 M_h'^{2}(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}} \qquad (18.2)$$

in the case of temperature dependence at given frequency, $\omega = \underline{\omega}$.
- The general expression of the *corresponding loss compliance* is, taking into account (2), (10) and (13), given as

$$J_M''(\omega,T) = \frac{[A_1(T) - A_0(T)B_1(T)]\omega}{A_0^{\;2}(T) + A_1^{\;2}(T)\omega^2} =$$

$$= \frac{[M_h'(T) - M_l'(T)]\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{T}}\omega}{M_l'^{2}(T) + M_h'^{2}(T)\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{T}}\omega^2} \qquad (19)$$

while for the ω - dependence at fixed temperature, $T = \underline{T}$,

$$J_M''(\omega;\underline{T}) = \frac{[A_1(\underline{T}) - A_0(\underline{T})B_1(\underline{T})]\omega}{A_0^{\;2}(\underline{T}) + A_1^{\;2}(\underline{T})\omega^2} = \frac{(A_{1\underline{T}} - A_{0\underline{T}}B_{1\underline{T}})\omega}{A_{0\underline{T}}^{\;2} + A_{1\underline{T}}^{\;2}\omega^2} =$$

$$= \frac{(M_{h\underline{T}}' - M_{l\underline{T}}')\tau_{\varepsilon\infty}e^{\frac{\theta_\varepsilon}{\underline{T}}}\omega}{M_{l\underline{T}}'^{\;2} + M_{h\underline{T}}'^{\;2}\tau_{\varepsilon\infty}^{\;2}e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2} \qquad (19.1)$$

and

$$J_M''(T;\underline{\omega}) = \underline{\omega}\frac{A_1(T) - A_0(T)B_1(T)}{A_0{}^2(T) + \underline{\omega}^2 A_1{}^2(T)} =$$

$$= \underline{\omega}\frac{[(M_h'(T) - M_l'(T)]\tau_{g\infty}e^{\frac{\theta_\varepsilon}{T}}}{M_l'^2(T) + \underline{\omega}^2 M_h'^2(T)\tau_{g\infty}{}^2 e^{\frac{2\theta_\varepsilon}{T}}} \qquad (19.2)$$

in the case of T - dependence at given frequency, $\omega = \underline{\omega}$.

- The general form of the *corresponding absolute compliance* results, given (2), (11) and (13), as

$$|J_M^*(\omega,T)| = [\frac{1 + B_1{}^2(T)\omega^2}{A_0{}^2(T) + A_1{}^2(T)\omega^2}]^{1/2} =$$

$$= [\frac{1 + \tau_{g\infty}{}^2 e^{\frac{2\theta_\varepsilon}{T}}\omega^2}{M_l'^2(T) + M_h'^2(T)\tau_{g\infty}{}^2 e^{\frac{2\theta_\varepsilon}{T}}\omega^2}]^{1/2} \qquad (20)$$

whereas for the frequency dependence at given temperature, T=T, one obtains

$$|J_M^*(\omega;\underline{T})| = [\frac{1 + B_1{}^2(\underline{T})\omega^2}{A_0{}^2(\underline{T}) + A_1{}^2(\underline{T})\omega^2}]^{1/2} = (\frac{1 + B_{1\underline{T}}{}^2\omega^2}{A_{0\underline{T}}{}^2 + A_{1\underline{T}}{}^2\omega^2})^{1/2} =$$

$$= (\frac{1 + \tau_{g\infty}{}^2 e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2}{M_{l\underline{T}}'^2 + M_{h\underline{T}}'^2\tau_{g\infty}{}^2 e^{\frac{2\theta_\varepsilon}{\underline{T}}}\omega^2})^{1/2} \qquad (20.1)$$

and

$$|J_M^*(T;\underline{\omega})| = [\frac{1 + \underline{\omega}^2 B_1{}^2(T)}{A_0{}^2(T) + \underline{\omega}^2 A_1{}^2(T)}]^{1/2} =$$

$$= (\frac{1 + \underline{\omega}^2 \tau_{g\infty}{}^2 e^{\frac{2\theta_\varepsilon}{T}}}{M_l'^2(T) + \underline{\omega}^2 M_h'^2(T)\tau_{g\infty}{}^2 e^{\frac{2\theta_\varepsilon}{T}}})^{1/2} \qquad (20.2)$$

in the case of temperature dependence at given frequency, $\omega = \underline{\omega}$.

The explicit analytical relations of the complete set of seven rheodynamical quantities reveal both the potential consequences - from the viewpoint of different forms of considered dependences - as well as the effectively ones - as a start up point for the identification of the independent values corresponding to characteristic - extremum (maximum and minimum) and inflection points.

Thus, in the case of *isothermal* circumstances, T=T, the frequency dependence at strain-controlled conditions shows:

- for the direct primary rheodynamical quantities - $M'(\omega;\underline{T}), M''(\omega;\underline{T})$ - that the ω - terms appearing in the characteristic ratios are of same order (2 to 2) in the case of M', whereas it is a dissimilar one, (1 to 2), for M'';

- for the derivate primary rheodynamical quantities - $|M^*(\omega,\underline{T})|, \beta_M(\omega,\underline{T})$ - the ratio is (2 to 2), in the case of $|M^*(\omega,\underline{T})|$, and a (1 to 2) one for $\beta_M(\omega,\underline{T})$;

- for the corresponding secondary rheodynamical quantities -, $-J_M'(\omega,\underline{T}), J_M''(\omega,\underline{T}), |J_M^*(\omega,\underline{T})|$ - the values of characteristic ratios are (2 to 2) in the case of $J_M'(\omega,\underline{T})$ and $|J_M^*(\omega,\underline{T})|$ while it is (1 to 2) for J"$_M(\omega, T)$.

On the other hand, in the case of *isochronal* circumstances, $\omega = \underline{\omega}$, at strain-controlled conditions, the temperature dependence points out:

- for the direct primary rheodynamical quantities, $M'(T;\underline{\omega}), M''(T;\underline{\omega})$, the T - exponentials appearing in the characteristic ratios reveals that in the case of M' the terms are of same order (2 to 2), but dissimilar ones (1 to 2) for M";

- for the derivate primary rheodynamical quantities - $|M^*(T;\underline{\omega})|, \beta_M(T;\underline{\omega})$ - the order of exponential terms which appear in the characteristic ratios is (2 to 2) in the case of $|M^*(T;\underline{\omega})|$ while the (1 to 2) result is obtained for $\beta_M(T;\underline{\omega})$

- for the corresponding secondary rheodynamical quantities - $J_M'(T;\underline{\omega}), J_M''(T;\underline{\omega}), |J_M^*(T;\underline{\omega})|$ - the characteristic ratios of exponential terms are given as (2 to 2) in the case of $J_M'(T;\underline{\omega})$ and $|J_M^*(T;\underline{\omega})|$ and (1 to 2) for $J_M''(T;\underline{\omega})$.

Conclusions

The explicit analytical relations obtained in the case of dynamic strain-controlled processes for both the primary rheological quantities as well as for the corresponding secondary ones tell us well definite qualitative features concerning the frequency or/and temperature dependences.

Moreover, it is revealed the natural contribution of frequency dependence in *isothermal* circumstances, whereas for the temperature dependence in *isochronal* circumstances well defined contribution of exponential terms containing the activation energy (virtual temperature) are present.

Furthermore, the direct thermorheodynamic quantities - the storage and absolute moduli - on the hand, as well as, on the other hand, the corresponding ones - the storage and absolute compliances - are typical in their frequency dependence, the situation being somewhat comparable from the standpoint of temperature dependence.

Henceforth, in the case of the other considered direct thermodynamic quantities, including the loss modulus and the loss facor, as well as for the corresponding loss compliance, the frequency or temperature trends appear to be similar.

Definite peculiarities of the full set of characteristic thermorheodynamic quantities are to be identified by taking into account distinct frequency or/and temperature dependences on the basis of numerical simulation route.

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